Houston, We Have a Problem!

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The Problem Solving Problem

“I have the answer, but I can’t explain how I got it!” —5th grade student

How you think about a problem is more important than the problem itself - so always think positively.

---Norman Vincent Peale
Instructional programs from prekindergarten through grade 12 should enable all students to—

• build new mathematical knowledge through problem solving;

• solve problems that arise in mathematics and in other contexts;

• apply and adapt a variety of appropriate strategies to solve problems;

• monitor and reflect on the process of mathematical problem solving.

• communicate their mathematical thinking coherently and clearly to peers, teachers, and others

(http://standards.nctm.org/document/chapter3/prob.htm)
Why Include Problem Solving?

• Provides opportunities for higher order thinking skills
• Solution or means to solving is not always apparent
• Originality or creativity required
• Requires more sophisticated cognitive skills than those required for basic computation

(George Lenchner, 1983)
George Lenchner was a mathematics teacher and the founder of Math Olympiads for Elementary and Middle Schools, a nonprofit educational organization. MOEMS sponsors programs and competitions for fourth through eighth graders. Mr. Lenchner earned a master's degree in music from New York University, a master's degree in math from Adelphi University in 1965, and a doctorate in math education from Columbia in 1972. He wrote several books, including "Creative Problem Solving in School Mathematics: A Handbook for Teachers, Parents, Students and Other Interested People."

Learning to Stay in the Struggle

*Outliers*, by Malcolm Gladwell

• TIMSS
  – Questionnaire Completion = Test Completion

• Puzzle Persistence
  – Japanese 1st Graders - 13.93 minutes
  – American 1st Graders - 9.47 minutes

• Story of Renee
It's not that I'm so smart, it's just that I stay with problems longer. --Albert Einstein

No problem can stand the assault of sustained thinking. –Voltaire

I used to think that, if I took longer than about 15 minutes on a problem, I must have been doing it wrong and I felt stupid. —student (Tretter)
Teaching For, About & Through Problem Solving

• *For*-format used by traditional math textbooks

• *About*-strategies, techniques, or processes

• *Through*-method of learning deep understanding of mathematics and concepts
Types of Problems

• Routine
  – Simple translation
  – one-step
  – multi-step

• Nonroutine
  – Puzzle
  – Project
  – Process
Levels of Demands

- Lower-level (memorization)
- Lower-level (procedures without connections)
- Higher-level (procedures with connections)
- Higher-level (doing mathematics)

--Selecting & Creating Mathematical Tasks: From Research to Practice
Memorization

• What is the divisibility rule for 3?

• $7 \times 8 =$?

• Identify the place value of the 3 in 43,679.
Procedures without Connections

• $427 \times 34 = ?$ Show work and check with a calculator.

• Answering questions about a bar graph such as “How many books were read the first week?”

• Multiply $\frac{4}{5}$ and $\frac{2}{3}$. 
Procedures with Connections

• About how long is $\frac{4}{5}$ of this rectangle? Shade your answer. What other fractions are near $\frac{4}{5}$ in size?

• Find $\frac{1}{6}$ of $\frac{1}{2}$. Use pattern blocks. Draw your answer, and explain your solution.
Kara is building a fence for her dog. She purchased 50 1-foot sections of fence. Show the best design for her fence, calculate the area, and justify your design.

Joe ate \( \frac{1}{2} \) of a pizza, and Raj ate \( \frac{1}{2} \) of another pizza. Raj said they ate the same amount, but Joe said he ate more. Use words and pictures to show that Joe could be correct.
Level?

• Use four 4’s and any math symbols to create number sentences that equal 1-20.
  Example: 4 \times 4 + 4/4 = 17

• Create a six-digit number divisible by 2, 3, 4, 5 and 6.
Level?

• Six arrows land on a target that consists of 3 concentric circles. The circles are worth 3, 5 or 7 points. Which of the following sums are possible?

  16, 19, 24, 36, 41, 44
Level?

- Jo made 5 of 8 free throws; Leila made 11 of 17; and Rhea made 7 of 11. Who should attempt the game-winning free throw? Explain your answer, and justify your choice.
George Polya (1887-1985)

- George Polya’s major contribution is for his work in problem solving. He developed a method of problem solving that is still respected today. Polya believed that the skill of problem solving was not an inborn quality but something that could be taught. He taught a short time at Brown University and much of his later life at Stanford University. He published books on mathematics and became known as the father of problem solving.

(http://www.math.wichita.edu/history/men/polya.html)
George Polya (1887-1985)

SEE, PLAN, DO, CHECK

• Understand the Problem - (SEE)
• Devise a plan - (PLAN)
• Carry out the plan - (DO)
• Check the answer - (CHECK)
The Process of Problem Solving

EARLY PHASES
- Analysis
- Representation
- Planning

MIDDLE PHASES
- Selecting Strategies
- Applying Strategies

LATE PHASES
- Evaluation of Progress on the Solution
- Review
- Monitor
- Persist
- Ask the Teacher (Interaction)

(Adelaide, 1988)
The Art of Teaching Problem Solving

• Solving problems is a practical art, like swimming, or skiing, or playing the piano: you can learn it only by imitation and practice. . . . . if you wish to learn swimming you have to go in the water, and if you wish to become a problem solver you have to solve problems.

  - Mathematical Discovery, George Polya
Dr. Polya’s Strategies

- Guess and Check
- Make an Organized List
- Draw a Picture or a Diagram
- Look for a Pattern
- Make a Table
- Use a Variable
- Solve a Simpler Problem
- Experiment
- Act It Out or Use Objects
- Work Backwards
- Use Deduction
- Change Your Point of View
Ideas for Instruction

• Present one strategy at a time
• Mix it up
• Integrate with other math topics
• Thorough and well-communicated explanations of the solutions
• Work in pairs, triads or groups
• Each student must write down the solution
The Greatest Challenge: *Thorough and well-communicated explanations of the solutions*

If a mathematician wants to contribute to the greater body of mathematical knowledge, she must be able to communicate her ideas in a way which is comprehensible to others. (Lee)

Writing only the “final answer” would be like Herman Melville writing only the single sentence: *The whale wins.* (Lee)
Metacognition
Thinking about Thinking!

• How did you decide to wear what you are wearing today?
Possibilities…

- Activities
- Weather
- Comfort
- Impression
- Accessories
- Clean
- Ironed
- Shoes
- Repeat
- Color
- Other?
Writing good mathematical explanations …

• Will improve knowledge and understanding of mathematics

• Requires careful thought and attention

• Means the mathematics is often more likely correct

• Helps students learn and retain concepts

(Lee)
Marilyn Burns (1941-)

- Marilyn Burns is the creator and founder of *Math Solutions Professional Development*, dedicated to the improvement of math instruction in Grades K–8. Ms. Burns has taught children, led in-service workshops, written professional development publications for teachers and administrators, created staff development videotapes, and written books for children.

(ftp://teacher.scholastic.com/products/paperbacks/marilynburns/bio.htm)
Marilyn Burns
Writing in Mathematics

• Purpose, logic and reasonableness

• Organize ideas; develop and extend learning

• Reflect on ideas

• Allows for assessment of thinking and understanding

• Encourage different ways of thinking

(Burns, 2005)
• In fact, merely writing the facts on a piece of paper and stating our problem clearly goes a long way toward helping us to reach a sensible decision. As Charles Kettering puts it: "A problem well stated is a problem half solved."

--Dale Carnegie
Personal Experiences in Problem Solving Instruction

• 10 strategies
  – Act It Out or Use Objects

• Three explorers and three cannibals wish to cross a river. There is a boat that can carry up to three people, and either explorers or cannibals can operate the boat. However, it is never permissible for the cannibals to outnumber the explorers either in the boat or on the shore.
The P.O.W.
Problem of the Week

Purpose: To provide students with opportunities to refine problem solving skills, communicate reasoning and strategies, and reflect upon their work.
Components of the POW Worksheet

• **Initial Response** (List the important information.)

• **What must you find?** (Restate the problem you are to solve or the question you are to answer.)

• **Strategies for Solving** (Circle all that you use.)
  
  Act It Out  
  Draw a Picture or Diagram  
  Make a List, Table or Chart  
  Guess and Check  
  Find a Pattern  
  Work Backward  
  Logical Reasoning  
  Write an Equation  
  Choose the Correct Operation  
  Solve a Simpler Problem

• **First Attempts** (Show/Describe how you first try to solve the problem—similar to a rough draft.)
• **Final Work and Solution** (Show the steps used to solve the problem. Organize your work.)

• **Detailed Explanation** (Explain how you finally solved the problem. Your writing should be completed in full sentences and it should include detail. You must explain why you did what you did and how you knew what you knew—because statements should be included. Use only the space provided.)

• **Reflection** (Discuss what worked, what didn’t work, and what you did when you got stuck. Did you get help from anyone? What kind of help? What relationships did you notice between the work you did for this problem and your work for other problems you have solved? How did your errors help you solve the problem? If you solved a similar problem, how would you solve it differently?)
Step-by-Step

- Graphic Organizer for Pre-writing
  What was done? Why was it done?

- Rubric
  Student revisions
  Student practice
  Two trials
  Collaboration
Completion of Work

- The entire worksheet is complete.
  * Important Information
  * What must be found?
  * Strategies?
  * First Attempts
  * Final Work & Detailed Explanation
The Work

- All steps necessary for solving are included. (5)
- Work is accurate. (4)
- Work is easy to understand; computation is labeled. (3)
- Final work is shown in step-by-step format. (2)
- Appropriate problem solving strategies are used. (2)
- Work demonstrates circled strategies (2)
- Work is neat and organized (2)
The Written Explanation

- *Why* you did what you did, and/or sometimes, *how* you knew what you knew, is explained. (8)
- All steps are adequately explained. (the *what*) (4)
- The mathematical reasoning is reasonable! (3)
- The explanation is mathematically accurate. (2)
- Mathematical vocabulary is used appropriately. (1)
- Writing is concise and focused; little “fluff.” (1)
- Writing is organized—beginning, middle & end. (1)
The Final Answer

- Final answer is correct. (4)
- Final answer is labeled correctly. (1)
An Example

- Alice is planning her garden in a square plot 6' x 6'. She wants to plant pepper plants one foot apart, in rows that are also one foot apart. She leaves a border one foot from each edge of the plot. How many pepper plants can she plant?
Resources for Problems

• MOEMS website and publications
• Ed Zaccaro books
• Brainteaser books and magazines
• WASL website
  http://www.portangelesschools.org/students/wasl.html
• Stusuggestions
• dent Connected Mathematics
• Teachers Pay Teachers
Other Problem Solving Activities

- Genius Cards
- Various Puzzles & Mindbenders
- Olympiad Activities
- Strategic-Thinking Games
- Competitions

Each problem that I solved became a rule, which served afterwards to solve other problems.

--Rene’ Descartes
Math Competitions

- National elementary school competitions (grades K-6)
- ABACUS International Math Challenge (grades 3-8)
- MATHCOUNTS (ages 9 -14)
- Continental Mathematics League (grades 2 - 9, Calculus and Computer)
- Kumon Math Challenge (grades 1-10)
- Math Kangaroo Competition (grades 2-12)
- Math League (grades 4-12)
- MOEMS (Olympiad) (grades 4-6 and 7-8)
- MathFax (grades 3-12)
- MathWizz (grades K-12)
- Monthly Math Challenge Problems (grades K-2)
- Noetic Learning Math Contest (grades 2-5)
- Online Math League (grades 2-9)
- SBU Math Contest (grades 3-8)
- The Math & Logic Contests (grades 2-6)

(Wikipedia.com)
More Math Competitions

- National middle school competitions (grades 6-8)
- ABACUS International Math Challenge (grades 3-8)
- MATHCOUNTS (ages 9 -14)
- American Mathematics Contest, formerly the American Junior High School Mathematics Examination (AJHSME)
- Continental Mathematics League (grades 2 - 9, Calculus and Computer)
- Kumon Math Challenge (grades 1-10)
- Math Kangaroo Competition (grades 2-12)
- Math League (grades 4-12)
- MOEMS (Olympiad) (grades 4-6 and 7-8)
- MathFax (grades 3-12)
- MathWizz (grades K-12)
- Online Math League (grades 2-9)
- SBU Math Contest (grades 3-8)
- Rocket City Math League(pre-algebra to calculus)
- Sigma Math League (grades 6-9)
- United States of America Mathematical Talent Search (USAMTS)

(Wikipedia.com)
Final Thoughts

"The problem is not that there are problems. The problem is expecting otherwise and thinking that having problems is a problem."

— Theodore Rubin
Sources and References


Additional websites cited:

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